A NEW ALGORITHM TO SOLVE ZERO ONE PROGRAMMING PROBLEM

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Abstract: When formulating linear programming problem, variables should have been regarded as taking integer values but for the sake of convenience, let them take fractional values and at the end any fractional part could be neglected. Whilst this is acceptable in some situations, in many cases it is not, and in such cases one must find a numeric solution in which the variables take integer values. Problems in which this is the case are called integer program and the subject of solving such programs is called integer programming. For solving integer programming problem no similar general purpose and computationally effective algorithms exist. Indeed theory suggests that not general-purpose computational complexity and concerns NP-completeness. It was developed from the early 1970's onward and basically is a theory concerning "how long it takes algorithms to run". This means that integer programmings are a lot harder to solve than linear programming. In this paper a new algorithm to solve zero-one integer linear programming problem if all the variables are restricted to take the values zero or one is given. This algorithm consists of two steps. In step 1 the intercepts of a promising variable based on the different constraints are found out. Using the intercept matrix obtained for all the promising variables, a maximum of m variables are selected and arranged where m is the number of constraints. Also the maximum value that each of the arranged variables can assume is found out. In step 2, the arranged all variables are allowed to enter into the basis simultaneously. A method has been suggested to find out the integer value restricted to zero or one with which a variable has to enter the basis. The simplex method while finding the improved basic feasible solution if moves along the edges of the feasible region. In the proposed zero-one integer linear programming algorithm the improved solution moves in the interior of the feasible region.

Keywords: Arrangement of variables, feasible solution, zero one algorithm

I. INTRODUCTION

Zero one Integer Linear Programming Problem is a special case of Integer Programming Problem. There are various methods used to solve such problems. To reduce the computational effort a new algorithms has been proposed to solve zero-one integer programming problem. In this algorithms the decision variables are arranged based on the maximum contribution to the objective function. The arranged variables are then allowed simultaneously to enter into the basis. This leads to reduce the computational time by means of reducing the iteration. The descriptions of new algorithm have been discussed as below.

II. STRUCTURE OF ZERO-ONE INTEGER LINEAR PROGRAMMING PROBLEM

In a integer programming problem, if all the variables are restricted to take the values either 0 or 1 only, then the given problem is known as zero-one programming problem.

The general zero-one integer programming problem can be stated as:

Extremize $Z = CX$ Subject to

$$AX \begin{matrix} < \\ = \\ > \end{matrix} P_0$$

and $X = 0$ or 1.

where

$$\underset{(mxn)}{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} ; \underset{(nx1)}{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} ; P_0 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

Let the columns corresponding to the matrix A be denoted by $P_1, P_2, \ldots.. P_n$ where

$$\underset{(mx1)}{P_1} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix} \text{ and } P_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

III. ZERO-ONE INTEGER LINEAR PROGRAMMING ALGORITHM

This algorithm consists of two phases. In first phase promising variables are arranged and in second phase arranged variables are allowed to enter into the basis simultaneously.

a) Phase I Arrangement of promising variables

Step 1. The matrix of intercepts of the decision variables along the respective axes called "θ" matrix with respect to the chosen basis is to be constructed. A typical intercept for the j^th variable, due to the i^th resource b_i is

$$\frac{b_i}{a_{ij}}, a_{ij} > 0$$

The expanded form of θ matrix is :

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \cdots \\ x_j \\ \cdots \\ x_n \end{array} \begin{array}{c} \begin{array}{ccccccc} s_1 & s_2 & \cdots & s_i & \cdots & s_m \end{array} \\ \begin{bmatrix} b_1/a_{11} & b_2/a_{21} & \cdots & b_i/a_{i1} & \cdots & b_m/a_{m1} \\ b_1/a_{12} & b_2/a_{22} & \cdots & b_i/a_{i2} & \cdots & b_m/a_{m2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_1/a_{1j} & b_2/a_{2j} & \cdots & b_i/a_{ij} & \cdots & b_m/a_{mj} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_1/a_{1n} & b_2/a_{2n} & \cdots & b_i/a_{in} & \cdots & b_m/a_{mn} \end{bmatrix} \end{array}$$

Each row of the θ matrix represents the m number of intercepts of the decision variables along their respective axes and the each column represents the intercepts formed by the decision variables of each of the m constraints.

Step 2. Scan each row of θ matrix and find the niminmum intercept and its position. Multiply the minimum intercepts with the corresponding contribution coefficient (c_j) value.

Step 3. Find the variable whose $c_j x_j$ value is the largest. Let it be x_R. Then x_R is selected as the promising variable. If more than one largest $c_j x_j$ value occurs, consider the variable that has maximum contribution coefficient including the fractional value is selected as promising variable. Delete the x_R^{th} row as well as the other rows whose minimum occurs in the position at which the minimum for x_R occurs. If more than one minimum occurs consider the variable that has minimum coefficient value including the fractional part. R is stored as the k^th element in set J, increment k by 1.

Step 4. Repeat step 3 till all the rows or all the columns are deleted and the rows and columns of the basic variable already entered are also to be deleted.

Step 5. The set of variables collected in step 4 are the ordered promising variables.

$$\text{Let } J = \left\{ \begin{array}{l} \text{Subscripts of the promising} \\ \text{variables arranged in the descending} \\ \text{order of } \; c_j x_j \; \text{ value} \end{array} \right\}$$

b) Phase II – Arranged variables are allowed to enter in to the basis simultaneously

Step 1. Initially P_0 value is assigned to $P_{0new.}$

Step 2. Select all the variables in the set J and allowed to enter into the basis with the value 1. That is j^th variable of x_R =1. Where j=1,2,…,k.

Step 3.

$$\text{Find } (P_{0new})_i = (P_{0old})_i - \sum_{z=1}^{k} a_{ij}\, x_{zj};$$

$$\text{Where } i = 1,2,...,m,\ j = 1,2,...,n \text{ and } k = 1,2,...,k$$

$$\text{Replace } (P_{0old})_i \text{ by } (P_{0new})_i$$

Step 4. Repeat Phase I and Phase II until $(P_{0new})_i \geq 0$

IV. NUMERICAL EXAMPLE

Max $Z = 20x_1 + 40x_2 + 20x_3 + 15x_4 + 30x_5$

Subject to

$$5x_1 + 4x_2 + 3x_3 + 7x_4 + 8x_5 \le 25$$

$$x_1 + 9x_2 + 9x_3 + 4x_4 + 6x_5 \le 25$$

$$8x_1 + 10x_2 + 2x_3 + x_4 + 10x_5 \le 25$$

Solution

To find θ matrix

$$\begin{pmatrix} 5 & 6.25 & 8.3 & 3.57 & 3.31 \\ 25 & 3.57 & 2.71 & 6.25 & 4.19 \\ 3.31 & 2.5 & 12.5 & 25 & 2.5 \end{pmatrix}$$

$$c_j x_j \rightarrow 60 \quad 80 \quad 40 \quad 45 \quad 60$$

Ordered set J = $\{x_2, \; x_4, \; x_3\}$

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\downarrow \qquad \downarrow$$

$$P_{0new} = \begin{pmatrix} 25 \\ 25 \\ 25 \end{pmatrix} \quad \begin{pmatrix} 11 \\ 5 \\ 12 \end{pmatrix}$$

θ matrix

$$\begin{pmatrix} 2.2 & 2.75 & 3.67 & 1.57 & 1.38 \\ 5 & 0.71 & 0.56 & 1.25 & 0.83 \\ 1.5 & 1.20 & 6 & 12 & 1.2 \end{pmatrix}$$

$$c_j x_j \rightarrow 20 \quad \ldots \quad \ldots \quad 15 \quad \ldots$$

x_4 already entered. So Ordered set J = x_1

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\downarrow \qquad \downarrow$$

$$P_{0new} = \begin{pmatrix} 11 \\ 5 \\ 12 \end{pmatrix} \quad \begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix}$$

Sol: $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1$ and $x_5 = 0$

$\therefore$ Max z = $20x_1 + 40x_2 + 20x_3 + 15x_4 + 30x_5$

= 20 x 1+ 40 x 1+ 20 x 1+ 15 x 1 +30 x 0

= 20 + 40 + 20 + 15 = 95

V. CONCLUSION

In this paper a new algorithm has been devised to solve zero-one integer linear programming problem. It is initially the promising variables are arranged based on the contribution to the objective function. Then the arranged variables are allowed to enter into the basis simultaneously. This process is executed until the constraint are satisfied. This algorithms has been used to solve zero-one integer programming problem with less than or equal to constraint and positive coefficient. This algorithm computational time has been reduced.

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